GCE

Further Mathematics A

Y540/01: Pure Core 1

A Level

Mark Scheme for June 2024

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. **Annotations**

Annotation	Meaning
✓and✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

5. **Subject Specific Marking Instructions**

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. ‘can’t do’, ‘don’t know’)
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn’t an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is **not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:

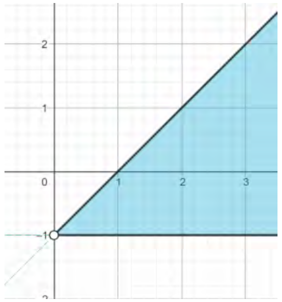
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question		Answer	Marks	AO	Guidance
1		$\frac{1}{\sqrt{(1-(x^2)^2}}$	B1	1.1	For $\frac{1}{\sqrt{(1-(x^2)^2}}$ seen.
		$\times 2x$	M1	1.1	For $2x \times \mathrm{f}(x)$ where $\mathrm{f}(x)=\frac{1}{\sqrt{1-(x^2)^2}}$ or $\frac{1}{\sqrt{1-x^2}}$ ONLY.
		$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{2x}{\sqrt{1-x^4}}$	A1	1.1	Allow any equivalent correct form e.g. $2x(1-x^4)^{-0.5}$. Must be in terms of *x*. Condone $(x^2)^2$ for x^4. ISW once correct answer seen.
			[3]		
		Alternative method			
		$\cos y\frac{\mathrm{d}y}{\mathrm{d}x}=2x$ or $\cos y=2x\frac{\mathrm{d}x}{\mathrm{d}y}$	B1		For correctly differentiating implicitly with respect to either *x* or *y*.
		$\sqrt{1-(x^2)^2}\frac{\mathrm{d}y}{\mathrm{d}x}=2x$	M1		Replacing $\cos y$ with $\pm\sqrt{\pm 1\pm(x^2)^2}$ in their derivative of the form $\pm\cos y\frac{\mathrm{d}y}{\mathrm{d}x}=2x$ (or equivalent if differentiating with respect to *y*).
		$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{2x}{\sqrt{1-x^4}}$	A1		Allow any equivalent correct form e.g. $2x(1-x^4)^{-0.5}$. Must be in terms of *x*. Condone $(x^2)^2$ for x^4. ISW once correct answer seen.
			[3]		

Question		Answer	Marks	AO	Guidance
2	(a)		M1	1.1	Half-line starting at one of $(-1,0),(1,0)$ $(0,1)$, or $(0,-1)$ at an angle of approx. $\frac{\pi}{4}$ to the positive horizontal. This mark can be awarded if the line is shown dashed rather than solid. This mark can be implied by shading that begins/ends where this line is meant to be even if the line is not explicitly shown.
			A1	1.1	Solid half-line starting at $(0,-1)$ passing through $(1,0)$ with region between half-line and $y=-1$ shaded. Neither line needs to be shown if the shading alone exactly defines the correct region. Condone dashed line for $y=-1$. **If shading the outside,** then must label the inside as C_1.
			[2]		
	(b)	$\arg(1.2+0.8\mathrm{i}+\mathrm{i})=\arctan\left(\frac{1.8}{1.2}\right)$	M1	1.1	Correct calculation for arg(1.2 + 1.8i) – allow $\tan\theta=\frac{1.8}{1.2}$ (oe) for **M1**.
		$=0.98...>\frac{\pi}{4}$ so no, $1.2+0.8\mathrm{i}$ is not in C_1.	A1	2.2a	Correct justification and conclusion. Evaluation of arctan must be given to at least **2** d.p. rot (0.982793…). For reference: $\frac{\pi}{4}=0.78(5398...)$. 'No' is sufficient as a conclusion. Award **A1** for both values (0.98… and 0.78…) together with correct conclusion but if $\frac{\pi}{4}$ not evaluated must see comparison with 0.98… e.g. $0.98...>\frac{\pi}{4}$
			[2]		

Question		Answer	Marks	AO	Guidance
		Alternative method			
		Cartesian equation of half-line is $y = x - 1$ so when $x = 1.2, y = ...$	M1		Substitutes $x = 1.2$ into $y = x - 1$
		$0.2 < 0.8$ so no, $1.2 + 0.8i$ is not in C_1.	A1		Correct justification and conclusion. Must see comparison of 0.2 and 0.8 for this mark. **SC B2** For stating that 1.8 > 1.2 or 1.5 > 1 and 'no' cwo.
			[2]		
	(c)		M1	1.1	For $\|z-3\|$ and 2 seen. Allow any letter for z. Allow use of $\text{mod}(z-3)$ for both marks.
		$\{z : \|z-3\| < 2\}$	A1	2.5	Set notation and inequality must be correct. Allow any letter for z. Condone $\{z : \|z-3\|^2 < 2^2\}$ **SC1** for $\{z : \|z+3\| < 2\}$ or $\{z : \|z-3\| < 4\}$
			[2]		
	(d)	$\|1.2 + 0.8i - 3\| = \sqrt{(-1.8)^2 + 0.8^2} \left(= \sqrt{3.24 + 0.64} = \sqrt{3.88}\right)$	M1	1.1	Calculating $\|1.2 + 0.8i \pm 3\|$ or $\|1.2 + 0.8i \pm 3\|^2$ correctly for their 3 (which must be real) from part **(c)**. Or for calculating $\|\pm(3-(1.2+0.8i))\|$.
		$\sqrt{3.88} < 2$, so yes, $1.2 + 0.8i$ is in C_2.	A1	2.2a	Comparing modulus with 2 (or modulus squared with 4) and concluding that 1.2 + 0.8i is contained in C_2. Allow just $\sqrt{3.88} < 2$ as a comparison. For reference $\sqrt{3.88} = 1.96(977...)$. Allow $\sqrt{3.88} \leqslant 2$. 'Yes' is sufficient as a conclusion. Allow other correct surds for comparison e.g. $\frac{\sqrt{97}}{5}$.
			[2]		

Question		Answer	Marks	AO	Guidance
3	(a)	For reference: $\mathbf{N}=\begin{pmatrix} a & 4 & 2 \\ 5 & 1 & 0 \\ 3 & 6 & 3 \end{pmatrix}$			
		$\mathbf{N}^2=\begin{pmatrix} a^2+26 & 4a+16 & 2a+6 \\ 5a+5 & 21 & 10 \\ 3a+39 & 36 & 15 \end{pmatrix}$	M1	1.1	A 3 by 3 matrix with at least three correct (simplified) entries.
			M1	1.1	A 3 by 3 matrix with at least two correctly simplified rows or at least two correctly simplified columns.
			A1	1.1	cao
			[3]		
	(b)	$\det\mathbf{N}=a\begin{vmatrix} 1 & 0 \\ 6 & 3 \end{vmatrix}-4\begin{vmatrix} 5 & 0 \\ 3 & 3 \end{vmatrix}+2\begin{vmatrix} 5 & 1 \\ 3 & 6 \end{vmatrix}$	M1	1.1	Correct method. May see $a(3-0)\ -4(15-0)+2(30-3)$. Ignore sign errors in 2×2 determinant calculations, but cofactors must have correct signs. Do look out for expanding by other rows and columns. If using Sarrus' method, must see correct 66 **and** $3a+60$. Or for $\begin{pmatrix} a \\ 5 \\ 3 \end{pmatrix}\bullet\left\|\begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}\times\begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}\right\|=\begin{pmatrix} a \\ 5 \\ 3 \end{pmatrix}\bullet\begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix}$ (oe) with at least one of $k_1=3, k_2=0, k_3=-2$ correct.
		$=3a-6$	A1	1.1	cao
			[2]		

Question		Answer	Marks	AO	Guidance
	(c)	11.6 × det **N**	**M1**	**1.1**	Or k × det **N** where $11.55 \leqslant k \leqslant 11.65$ and $12.5 \leqslant a \leqslant 13.5$ in their det **N** from part **(b)**. If not calculating the correct UB and LB (from a correct determinant) then we must see **either** a correct calculation (for their determinant from part **(b)**) **or** they must state, the values they've used in their calculation for both a and the volume of S_1 to award any marks.
		For example, Upper bound is 11.65 × (3×13.5 – 6) = 401.925 Lower bound is 11.55 × (3×12.5 – 6) = 363.825	**A1**	**3.1a**	For either correct upper **or** lower bound, **or any correct** V_2 **for any** value of a between 12.5 and 13.5, and $11.55 < V_1 < 11.65$. Allow answers rounded or truncated to the nearest integer or greater degree of accuracy.
		So, the student's claim (that the volume is less than 400) is not necessarily true.	**A1**	**2.2b**	Demonstrates that there are values of $12.5 \leqslant a < 13.5$ and $11.55 \leqslant k < 11.65$ such that both $V > 400$, and $V < 400$ and concludes that the volume may be greater or less than 400 (depending on a more accurately determined value of a and the volume of S_1). Conclusion must indicate that the claim could be correct but not necessarily so (oe). If **M0** then **SC1** for LB = 144.375 **and** UB = 157.275 (using 13 for det **N**).
			[3]		

Question	Answer	Marks	AO	Guidance
4	**DR** $\alpha\beta\gamma = -(-\frac{3}{2})\ \ (=\frac{3}{2})$	M1	1.1	For $\alpha\beta\gamma = -(-\frac{3}{2})$.
	$w = \frac{3}{2}x$	M1	3.1a	Appropriate substitution of the form $w = (\alpha\beta\gamma)x$ with **their** value of $\alpha\beta\gamma$ (need not be substituted into given cubic for this mark). Condone reciprocal e.g. $x = \frac{3}{2}w$.
	$2\left(\frac{2}{3}w\right)^3 + 3\left(\frac{2}{3}w\right)^2 + 6\left(\frac{2}{3}w\right) - 3(=0)$ $16w^3 + 36w^2 + 108w - 81 = 0$	A1	2.2a	Allow any integer multiple but must have integer coefficients. Must be an equation (so must = 0 **or** terms on both sides of an equation) with all terms simplified. Condone if in terms of *x*.
		[3]		
	Alternative method $\sum\alpha = -\frac{3}{2}, \quad \sum\alpha\beta = 3, \quad \alpha\beta\gamma = -\left(-\frac{3}{2}\right)$	M1		For at least one of $\alpha\beta\gamma, \alpha+\beta+\gamma, \alpha\beta+\beta\gamma+\gamma\alpha$ correct. Can be implied by one correct coefficient in new cubic equation.
	$\alpha\beta\gamma(\alpha+\beta+\gamma) = -\frac{9}{4}$ $(\alpha\beta\gamma)^2(\alpha\beta+\beta\gamma+\gamma\alpha) = \frac{27}{4}$ $(\alpha\beta\gamma)^4 = \frac{81}{16}$	M1		For at least two of $(\alpha\beta\gamma)^4$, $(\alpha\beta\gamma)^2(\alpha\beta+\beta\gamma+\gamma\alpha), \alpha\beta\gamma(\alpha+\beta+\gamma)$ correct – not from incorrect values of $\alpha\beta\gamma, \alpha+\beta+\gamma, \alpha\beta+\beta\gamma+\gamma\alpha$.
	$w^3 + \frac{9}{4}w^2 + \frac{27}{4}w - \frac{81}{16} = 0$ $16w^3 + 36w^2 + 108w - 81 = 0$	A1		Allow any integer multiple but must have integer coefficients. Must be an equation (so must = 0 **or** terms on both sides of an equation) with all terms simplified. Condone if in terms of *x*.
		[3]		

Qn	Answer	Marks	AO	Guidance
5	$\dfrac{12x^3}{(2x+1)(2x^2+1)} = A + \dfrac{B}{2x+1} + \dfrac{Cx+D}{2x^2+1}$	**B1**	1.1	Correct form (possibly implied by correct identity).
	$12x^3 \equiv A(2x+1)(2x^2+1) + B(2x^2+1) + (Cx+D)(2x+1)$	**M1***	1.1	Identity without fractions. Follow through their partial fraction expression with one denominator of $2x+1$ and the other with $2x^2+1$ - both numerators must contain at least a constant unknown. Some examples for **M1** below: $\dfrac{B}{2x+1}+\dfrac{Cx+D}{2x^2+1}$ so $12x^3 \equiv B(2x^2+1)+(Cx+D)(2x+1)$ $A+\dfrac{B}{2x+1}+\dfrac{C}{2x^2+1}$ so $12x^3 \equiv A(2x+1)(2x^2+1)+B(2x^2+1)+C(2x+1)$ $\dfrac{B}{2x+1}+\dfrac{C}{2x^2+1}$ so $12x^3 \equiv B(2x^2+1)+C(2x+1)$. $\dfrac{A}{2x+1}+\dfrac{Bx^2+Cx+D}{2x^2+1}$ so $12x^3 \equiv A(2x^2+1)+(Bx^2+Cx+D)(2x+1)$.
	For example: Equating coefficients of $x^3 : A = 3$ Let $x = 0$, gives $A + B + D = 0$	**M1dep***	1.1	Equates coefficients or substitutes to find an equation involving only their unknowns. Do not award this mark if only two unknowns in their partial fractions (so must be at least three unknowns (or implied unknowns)).
		A1	1.1	Any two ($A = 3$, $B = –1$, $C = –2$, $D = –2$) unknowns correct from a correct partial fraction form.
	$3 - \dfrac{1}{2x+1} - \dfrac{2x+2}{2x^2+1}$	**A1**	1.1	All four unknowns correct – condone stating the correct form of the partial fractions anywhere together with all four unknowns correctly stated without necessarily bringing both parts together as a single expression at the end.
		[5]		

	Alternative method			
	Constant term of (A =) 3	**B1**		By polynomial division or inspection.
	$\dfrac{-6x^2-6x-3}{(2x+1)(2x^2+1)} \equiv \dfrac{B}{2x+1}+\dfrac{Cx+D}{2x^2+1}$ and so $-6x^2-6x-3 \equiv B(2x^2+1)+(Cx+D)(2x+1)$	**M1***		Re-writing $\dfrac{\mathrm{f}(x)}{(2x+1)(2x^2+1)}$, where f($x$) is quadratic, as $\dfrac{B}{2x+1}+\dfrac{Cx+D}{2x^2+1}$ or $\dfrac{B}{2x+1}+\dfrac{C}{2x^2+1}$ **and** correct identity not involving fractions following through their partial fractions and quadratic f(x).
	For example: Equating coefficients of $x^2 : 2B+2C=-6$ Let x = 0, gives $B+D=-3$	**M1dep***		Equates coefficients or substitutes to find an equation involving only their unknown(s).
		A1		Any (B = –1, C = –2, D = –2) one unknown correct from a correct partial fraction form (so must have had a correct f(x)).
	$3-\dfrac{1}{2x+1}-\dfrac{2x+2}{2x^2+1}$	**A1**		All unknowns correct – condone stating the correct form of the partial fractions anywhere together with all four unknowns correctly stated without necessarily bringing both parts together as a single expression at the end. So must see correct partial fraction expression or $A+\dfrac{B}{2x+1}+\dfrac{Cx+D}{2x^2+1}$ stated and all correct values of A, B, C and D seen.
		[5]		

Question	Answer	Marks	AO	Guidance
6	**DR**			
	$\int \frac{18}{x^2\sqrt{x}}\,dx = 18\left(-\frac{2}{3}x^{-\frac{3}{2}}\right)(+c)$	**M1***	**1.1**	For obtaining $ax^{-\frac{3}{2}}$ where $a \neq 0$.
	$= 18\lim_{k\to\infty}\left(-\frac{2}{3}k^{-\frac{3}{2}} - \left(-\frac{2}{3}\times 9^{-\frac{3}{2}}\right)\right)$	**M1**	**1.1**	Correct use of 9 as a lower limit and any letter (except *x*) for the upper limit (so must be considering a finite upper limit) in their integrated expression (indicated by their power increased by 1). Need not see mention of limiting process for this mark.
	$k^{-\frac{3}{2}} \to 0$ as $k \to \infty$	**B1dep***	**2.1**	Taking limit as $k \to \infty$ for their expression of the form $ax^{-\frac{3}{2}}$ (so $\frac{1}{\sqrt{\infty^3}} = 0$ oe is **B0**). Implied by, for example, $\lim_{k\to\infty}\left[-\frac{2}{3}k^{-\frac{3}{2}} - \ldots\right] = 0 - \ldots$ but not, for example, for $-\frac{2}{3}k^{-\frac{3}{2}} - \ldots = 0 - \ldots$ without clear use of limiting process.
	$= \frac{4}{9}$	**A1**	**2.2a**	cao from correct integrated expression and finite upper limit (so dependent on both previous **M** marks but not the **B** mark). Accept equivalent **exact** forms e.g. $\frac{12}{27}$.
		[4]		

Question		Answer	Marks	AO	Guidance
7	(a)	$(2\sinh u\cosh u \equiv)2\left(\frac{e^u - e^{-u}}{2}\right)\left(\frac{e^u + e^{-u}}{2}\right)$	M1	1.2	Use correct definitions of cosh u and sinh u in terms of exponentials. Condone omission of $2\sinh u\cosh u$ for this mark. (Note $2\sinh u\cosh u$ **or** *LHS* must be included for subsequent **A1**.) **M1** can be implied for stating that $2\sinh u\cosh u \equiv \frac{1}{2}(e^{-u} - e^{-u})(e^u + e^{-u})$ (but not for the **A** mark).
		$2\sinh u\cosh u \equiv 2\left(\frac{e^u - e^{-u}}{2}\right)\left(\frac{e^u + e^{-u}}{2}\right)$ $\equiv \frac{1}{2}\left(e^{2u} + 1 - 1 - e^{-2u}\right) \equiv \frac{1}{2}\left(e^{2u} - e^{-2u}\right) \equiv \sinh 2u$	A1	2.1	Complete proof with no errors. Allow *LHS* and *RHS* in place of 2 sinh u cosh u and sinh 2u respectively. Minimally acceptable proof is of the form $RHS \equiv 2\left(\frac{e^u - e^{-u}}{2}\right)\left(\frac{e^u + e^{-u}}{2}\right) \equiv \frac{e^{2u} - e^{-2u}}{2} \equiv LHS$ in either direction. Do not condone sin for sinh or cos for cosh or x for u (unless "let $x = u$" stated) for **A1**. If candidate shows that both *LHS* and *RHS* are equal to some third expression, then a conclusion must be clearly stated for the **A1** mark. **SC B1** If candidate works with given identity and concludes, for example, that 1 = 1.
			[2]		
		Alternative method (working left to right) $(\sinh 2u \equiv)\frac{1}{2}\left(e^{2u} - e^{-2u}\right)$	M1		Use definition of $\sinh 2u$ in terms of exponentials. Condone omission of $\sinh 2u$ for this mark. (Note $\sinh 2u$ or *RHS* must be included for subsequent **A1**).
		$\sinh 2u \equiv \frac{1}{2}(e^{2u} - e^{-2u}) \equiv \frac{1}{2}(e^u - e^{-u})(e^u + e^{-u})$ $\equiv 2\times\frac{1}{2}(e^u - e^{-u})\times\frac{1}{2}(e^u + e^{-u}) \equiv 2\sinh u\cosh u$	A1		Complete proof must see $2\times\frac{1}{2}(e^u - e^{-u})\times\frac{1}{2}(e^u + e^{-u})$ before stating correct *RHS*.
			[2]		

Question		Answer	Marks	AO	Guidance
	(b)	For reference: $y=16\cosh x-\sinh 2x$ $\frac{\mathrm{d}y}{\mathrm{d}x}=16\sinh x-2\cosh 2x$	M1*	1.1	Differentiating to get $\pm16\sinh x\pm k\cosh 2x$ for some $k\neq0$. May be in exponential form $\frac{\mathrm{d}y}{\mathrm{d}x}=8\left(\mathrm{e}^{x}-\mathrm{e}^{-x}\right)-(\mathrm{e}^{2x}+\mathrm{e}^{-2x})$ - if using exponential form then the derivative must be of the form $\frac{\mathrm{d}y}{\mathrm{d}x}=\pm8\left(\mathrm{e}^{x}-\mathrm{e}^{-x}\right)\pm k(\mathrm{e}^{2x}+\mathrm{e}^{-2x})$ for some $k\neq0$.
		$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=16\cosh x-4\sinh 2x$	A1	1.1	Correct second derivative, condone un-simplified. May be in exponential form. $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=8(\mathrm{e}^{x}+\mathrm{e}^{-x})-2(\mathrm{e}^{2x}-\mathrm{e}^{-2x})$ (oe).
		$=16\cosh x-4(2\sinh x\cosh x)$	M1dep*	3.1a	Correct use of the result from part **(a)** in their second derivative or their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=8(\mathrm{e}^{x}+\mathrm{e}^{-x})-2(\mathrm{e}^{x}-\mathrm{e}^{-x})(\mathrm{e}^{x}+\mathrm{e}^{-x})$ using difference of two squares (oe e.g. setting equal to zero and converting to a quartic in e^{x} which if correct is $\mathrm{e}^{4x}-4\mathrm{e}^{3x}-4\mathrm{e}^{x}-1=0$). **If converting to exponentials at any point, then correct exponential expression(s) for sinh and cosh must be used.**
		$=8\cosh x(2-\sinh x)$			Or in exponential form $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2(\mathrm{e}^{x}+\mathrm{e}^{-x})(4-\mathrm{e}^{x}+\mathrm{e}^{-x})$
		$\cosh x>0\Rightarrow x=\sinh^{-1}2$ or $\sinh x=2$	A1	2.2a	Must state $\cosh x>0$ **or** $\cosh x\geqslant1$ **or** show that there is no solution of the equation $\cosh x=0$ (oe e.g. 'cosh cannot be 0', 'cosh is at least 1', 'cosh has a minimum at (0, 1)', '$\cosh^{-1}$ only valid for $x\geqslant1$' but **not** just 'cosh x = 0 has no solutions') - allow stating that the only solution satisfies the equation $\sinh x=2$. If using exponentials then must show clearly that there is only one solution of the correct equation (which for reference is $\ln(2+\sqrt{5})$).
			[4]		

Question		Answer	Marks	AO	Guidance
		Alternative method for second and third marks			
		$\frac{dy}{dx} = 16\sinh x - 2(\cosh^2 x + \sinh^2 x)$	M1dep*		Use of $\cosh 2x = \cosh^2 x + \sinh^2 x$ in their first derivative. $\frac{dy}{dx} = 8(e^x - e^{-x}) - \frac{1}{2}(e^x + e^{-x})^2 - \frac{1}{2}(e^x - e^{-x})^2$
		$\frac{d^2y}{dx^2} = 16\cosh x - 4\sinh x\cosh x - 4\sinh x\cosh x$	A1		Correct second derivative, condone un-simplified. May use exponential form: $\frac{d^2y}{dx^2} = 8(e^x + e^{-x}) - (e^x + e^{-x})(e^x - e^{-x}) - (e^x - e^{-x})(e^x + e^{-x})$
		Alternative method for first three marks			
		$y = 16\cosh x - 2\sinh x\cosh x$	B1		Using the result from part **(a)**. Or $y = 8(e^x + e^{-x}) - 0.5(e^x - e^{-x})(e^x + e^{-x})$
		$\frac{dy}{dx} = \sinh x\,(16 - 2\sinh x) - 2\cosh^2 x$	M1		Differentiating to get $\pm 16\sinh x \pm k\cosh^2 x \pm k\sinh^2 x$ or $\frac{dy}{dx} = \pm 8(e^x - e^{-x}) \pm k(e^x + e^{-x})^2 \pm k(e^x - e^{-x})^2$ for some $k \neq 0$.
		$\frac{d^2y}{dx^2} = 16\cosh x - 4\sinh x\cosh x - 4\sinh x\cosh x$	A1		Correct second derivative, condone un-simplified. May use exponential form: $\frac{d^2y}{dx^2} = 8(e^x + e^{-x}) - (e^x + e^{-x})(e^x - e^{-x}) - (e^x - e^{-x})(e^x + e^{-x})$

Question		Answer	Marks	AO	Guidance
	(c)	$x = \ln(2+\sqrt{5})$	B1	2.1	Condone $\ln\|2+\sqrt{5}\|$.
		$\cosh(\ln(2+\sqrt{5})) = \frac{1}{2}\left(2+\sqrt{5}+\frac{1}{2+\sqrt{5}}\right)(=\sqrt{5})$ or $\cosh(\ln(2+\sqrt{5})) = \sqrt{1+2^2}$ $(=\sqrt{5})$	B1FT	2.4	Finding the value of cosh x in a form not involving logs or exponentials following through **their** x of the form $\ln(a+\sqrt{b})$ where a is non-zero and $b > 0$ and $a+\sqrt{b} > 0$ – allow un-simplified. The correct y-coordinate implies this and the next **B** mark. For reference: $\cosh(\ln(a+\sqrt{b})) = \frac{1}{2}\left(a+\sqrt{b}+\frac{1}{a+\sqrt{b}}\right)$. Or from using $\cosh^2 x - \sinh^2 x \equiv 1$ with their value of $\sinh x$ from part **(b)**.
		$y = 12\sqrt{5}$	B1	2.4	Need not be stated as a coordinate. Accept any exact one term equivalent.
			[3]		

Question	Answer	Marks	AO	Guidance
8	Base case: when $n=0,\ 11\times7^0-13^0-1=9$ which is divisible by 3.	B1	2.5	Base case shown to be true – condone when $n = 0$, 11 – 1 – 1 = 9 or just 9, must say that 9 is divisible by 3 or show explicitly e.g. $9=3\times3$. Allow base case with $n = 1$, $11\times7-13-1=63$ or just 63, must say that 63 is divisible by 3 or show explicitly e.g. $63=3\times21$.
	Assume that, when $n=k, 11\times7^n-13^n-1$ is divisible by 3. That is, $11\times7^k-13^k-1=3m$ (for some integer m).	M1*	2.1	Forms correct inductive hypothesis as an equation in terms of two different letters (oe in words e.g. $11\times7^k-13^k-1$ is divisible by 3) – condone using n e.g. $11\times7^n-13^n-1=3m$ for this mark.
	$11\times7^{k+1}-13^{k+1}-1$ $=7\times(3m+13^k+1)-13^{k+1}-1$	M1dep*	3.1a	Considers $11\times7^{k+1}-13^{k+1}-1$ and use inductive hypothesis that $11\times7^k-13^k-1=3m$ - must not be using n for k.
	$=3(7m+2-2\times13^k)$ (which is divisible by 3).	A1	2.2a	Shows using the inductive hypothesis that $11\times7^{k+1}-13^{k+1}-1$ is a multiple of 3. Some common answers are $3(-22\times7^k+4+13m)$ or $3(7m+2-2\times13^k)$ or $3(m+22\times7^k-4\times13^k)$.
	So, if true for $n = k$ this implies true for $n = k + 1$. True when $n = 0$ so therefore true for all integers $n\geqslant0$.	A1	2.4	Conclusion, dependent on **B1M1M1A1** and no errors seen in their proof. Do not award this mark if $n = 0$ not used as base case. However, this mark can be awarded if $n = 1$ used as base case and $n = 0$ considered separately. Must mention, '$n = k$ **or** P(k)', '$n = k + 1$ **or** P($k + 1$)', '$n = 0$ **or** base case **or** P(0)' (provided P(n) is defined) and '$n\geqslant0$ **or** all n oe' condone 'all integers' but not 'positive integers' or similar incorrect statement.
		[5]		

	Alternative method Base case: when $n = 0$, $11 \times 7^0 - 13^0 - 1 = 9$ which is divisible by 3.	B1		Base case shown to be true – condone when $n = 0$, $11 - 1 - 1 = 9$ or just 9, must say that 9 is divisible by 3 or show explicitly e.g. $9 = 3 \times 3$. Allow base case with $n = 1$, $11 \times 7 - 13 - 1 = 63$ or just 63, must say that 63 is divisible by 3 or show explicitly e.g. $63 = 3 \times 21$.
	Assume that, when $n = k$, $11 \times 7^n - 13^n - 1$ is divisible by 3. That is, $\mathrm{f}(k) = 11 \times 7^k - 13^k - 1$ is divisible by 3.	M1*		Forms correct inductive hypothesis - condone using n e.g. $\mathrm{f}(n) = 11 \times 7^n - 13^n - 1$ for this mark. Do not need to use f(k) – can just state that $11 \times 7^k - 13^k - 1$ is divisible by 3.
	$\mathrm{f}(k+1) - \mathrm{f}(k) = (11 \times 7^{k+1} - 13^{k+1} - 1) - (11 \times 7^k - 13^k - 1)$ $= 6 \times 11 \times 7^k - 12 \times 13^k$	M1dep*		Considers correct expression for $\mathrm{f}(k+1) - \mathrm{f}(k)$ and simplifies to an expression of the form $\pm a \times 7^k \pm b \times 13^k$ - must not be using n for k.
	$\mathrm{f}(k+1) = \mathrm{f}(k) + 3(22 \times 7^k - 4 \times 13^k)$ (which is divisible by 3).	A1		Shows using the inductive hypothesis that $\mathrm{f}(k+1)$ is divisible by 3 so must be considering an expression for $\mathrm{f}(k+1)$ and not just an expression for $\mathrm{f}(k+1) - \mathrm{f}(k)$.
	So, if true for $n = k$ this implies true for $n = k + 1$. True when $n = 0$ so therefore true for all integers $n \geqslant 0$.	A1		Conclusion, dependent on **B1M1M1A1** and no errors seen in their proof. Do not award this mark if $n = 0$ not used as base case. However, this mark can be awarded if $n = 1$ used as base case and $n = 0$ considered separately. Must mention, '$n = k$ **or** P(k)', '$n = k + 1$ **or** P($k + 1$)', '$n = 0$ **or** base case **or** P(0)' (provided P(n) is defined) and '$n \geqslant 0$ **or** all n oe' condone 'all integers' but not 'positive integers' or similar incorrect statement.
		[5]		

Question		Answer	Marks	AO	Guidance
9	(a)	$(\ln(1+x))^2 = \left(x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots\right)^2$	M1	3.1a	Squaring the correct result for $\ln(1+x)$ with at least the first three terms.
		$= x^2 - x^3 + \ldots$	B1	1.1	First two terms correct.
		$\ldots + \frac{11}{12}x^4 + \ldots$	A1	2.2a	Third term correct allow any equivalent fraction or exact decimal equivalent. ISW and ignore higher power terms.
			[3]		
		Alternative method			
		$\mathrm{f}(0) = 0$ and $\mathrm{f}'(x) = \frac{2\ln(x+1)}{x+1}$	M1		For f(0) = 0 and correct first derivative (or stating that the second derivative is zero too).
		$\mathrm{f}'(0)=0,\ \mathrm{f}''(0) = 2,\ \mathrm{f}'''(0) = -6$	A1		Correct values for first, second and third derivatives For reference: $\mathrm{f}''(x) = \frac{2-2\ln(x+1)}{(x+1)^2}$ and $\mathrm{f}'''(x) = \frac{2(-3+2\ln(x+1))}{(x+1)^3}$ and $\mathrm{f}^{(4)}(x) = \frac{22-12\ln(x+1)}{(x+1)^4}$ and $\mathrm{f}^{(4)}(0) = 22$.
		$(\ln(1+x))^2 = x^2 - x^3 + \frac{11}{12}x^4 + \ldots$	A1		All terms correct (allow simplified or exact decimal equivalents for coefficients). ISW and ignore higher power terms.
			[3]		

Question		Answer	Marks	AO	Guidance
9	(b)	**DR**			
		$x^2 - x^3 + \frac{11}{12}x^4 + ... = 2x^3$	M1*	3.1a	Setting their expansion from part **(a)** equal to $2x^3$. Their expansion must have at least three terms with non-zero quadratic, cubic and quartic terms.
		$x^2\left(1 - 3x + \frac{11}{12}x^2\right) = 0$	M1dep*	1.1	Rearranging and factorising (allow sign slips only). Ignore higher order terms if the factor of x^2 is seen – their expansion must have contained no constant or linear terms. Stating their quadratic $1 - 3x + \frac{11}{12}x^2 = 0$ is fine for this mark. Stating either non-zero root of the correct (or their) quadratic can imply this mark.
		$x = 0.37(6690...)$ or $2.89(6036...)$ but $x = 0.38$ as expansion is only valid for $-1 < x \leqslant 1$.	A1	3.2a	Selects the smallest non-zero root (so must see both correct roots either exact $\frac{18 \pm 8\sqrt{3}}{11}$ or to at least **2** decimal places) and justifies with interval for validity being $-1 < x \leqslant 1$. Allow awrt 0.38 or $\frac{18 - 8\sqrt{3}}{11}$. Condone any indication that expansion is only valid between values of -1 and 1 or for values less than, or less than or equal, to 1. Note that including x^5 terms in expansion gives $x = 0.35997$ and including x^5 and x^6 terms in expansion give $x = 0.36504$. Ignore any attempt to work out corresponding y-coordinate.
			[3]		

Question		Answer	Marks	AO	Guidance
10	(a)	$F = ma$ so So $-(15\sin 4t + 6v\tan 2t) = 3\frac{\mathrm{d}v}{\mathrm{d}t}$	M1	3.3	Using $F = 3a$ and $a = \frac{\mathrm{d}v}{\mathrm{d}t}$ to form a differential equation - allow minor slips or sign errors but intention must be clear. Their F must be two terms only.
		$\frac{\mathrm{d}v}{\mathrm{d}t} + 2v\tan 2t = -5\sin 4t$ (so $P(t) = 2\tan 2t$ and $Q(t) = -5\sin 4t$)	A1	1.1	The correct differential equation in the form $\frac{\mathrm{d}v}{\mathrm{d}t} + P(t)v = Q(t)$ can imply this mark. $P(t)$ and $Q(t)$ do not need to be explicitly stated. ISW once correct form seen. If not written in the form, $\frac{\mathrm{d}v}{\mathrm{d}t} + P(t)v = Q(t)$ then $P(t)$ and $Q(t)$ must be explicitly stated.
			[2]		

Question		Answer	Marks	AO	Guidance
	(b)	$\mathrm{I}(t) = \mathrm{e}^{\int 2\tan 2t\,\mathrm{d}t}$	M1*	1.1	For $\mathrm{I}(t) = \mathrm{e}^{\int P(t)\mathrm{d}t}$ for their $P(t)$
		$= \mathrm{e}^{-\ln(\cos 2t)} \quad (= \sec 2t)$	M1	1.1	For $\mathrm{I}(t) = \mathrm{e}^{\pm k\ln(\cos 2t)}$ or $\mathrm{e}^{\pm k\ln(\sec 2t)}$ or $\pm k\cos 2t$ or $\pm k\sec 2t$ or $\pm a\cos^k 2t$ or $\pm a\sec^k 2t$ for $a, k \neq 0$.
		$\left(\frac{\mathrm{d}v}{\mathrm{d}t} + 2v\tan 2t\right)\times \mathrm{I}(t) = -5\sin 4t \times \mathrm{I}(t) \Rightarrow \frac{\mathrm{d}}{\mathrm{d}t}(v \times \mathrm{I}(t)) = -5\sin 4t \times \mathrm{I}(t)$			
		$v \times \sec 2t = -5\int(\sin 4t \times \sec 2t)\mathrm{d}t$	M1dep*	1.1	For $v \times \mathrm{I}(t) = k_1\int \sin 4t \times \mathrm{I}(t)\,\mathrm{d}t$ with their $\mathrm{I}(t)$ (in **any** form) and $k_1 \neq 0$.
		$(v\sec 2t =) -10\int \frac{\sin 2t\cos 2t}{\cos 2t}\mathrm{d}t = -10\int \sin 2t\,\mathrm{d}t$	M1	1.1	For simplifying *RHS* to $k_2\int \sin 2t\,\mathrm{d}t$ for any $k_2 \neq 0$ - dependent on all previous **M** marks.
		$v\sec 2t = 5\cos 2t\ (+c)$	A1	1.1	For correct general solution (any equivalent form). Condone lack of $+c$.
		$v(0) = 4.5 \Rightarrow c = -0.5$	M1	3.3	Using $v(0) = 4.5$ to find constant term – dependent on first three **M** marks and an attempt at integration.
		$0 = 5\cos 2t - 0.5 \Rightarrow \cos 2t = 0.1$	M1dep*	3.4	For a two-term equation of the form $\cos 2t = k_3$ where $\lvert k_3\rvert < 1$ and $k_3 \neq 0$ - dependent on all previous **M** marks.
		So, *B* stationary after 0.735 seconds.	A1	2.2a	Ignore if any other solution(s) found. Allow awrt 0.735 – for reference 0.7353144… 42.1 seconds (calculated in degrees) scores **A0**.
			[8]		

Question		Answer	Marks	AO	Guidance
11	(a)	$\mathbf{r}=\begin{pmatrix}0\\11\\7\end{pmatrix}+\dots$ or $\mathbf{r}=\begin{pmatrix}18\\2\\16\end{pmatrix}+\dots$ or $\mathbf{r}=\dots+\mu\begin{pmatrix}2\\-1\\1\end{pmatrix}$ or $\dots\begin{pmatrix}0\\11\\7\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\1\end{pmatrix}$	**B1***	**3.3**	For two correct components – the components are • $\mathbf{r}=\dots$ or $\vec{r}=\dots$ ONLY • position vector, • direction vector in lowest terms with parameter.
		$\mathbf{r}=\begin{pmatrix}0\\11\\7\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\1\end{pmatrix}$	**B1dep***	**3.3**	For a correct equation with direction vector in lowest terms. Condone lack of (or incorrect) range of values for parameter. Other common answers are: $\mathbf{r}=\begin{pmatrix}0\\11\\7\end{pmatrix}+\mu\begin{pmatrix}-2\\1\\-1\end{pmatrix}$, $\mathbf{r}=\begin{pmatrix}18\\2\\16\end{pmatrix}+\mu\begin{pmatrix}-2\\1\\-1\end{pmatrix}$, $\mathbf{r}=\begin{pmatrix}18\\2\\16\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\1\end{pmatrix}$, There are other possible answers, e.g. $\mathbf{r}=\begin{pmatrix}6\\8\\10\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\1\end{pmatrix}$,
			[2]		

Question		Answer	Marks	AO	Guidance
	(b)	$\begin{pmatrix}6\\7\\1\end{pmatrix}\times\begin{pmatrix}2\\-1\\1\end{pmatrix}=\begin{pmatrix}8\\-4\\-20\end{pmatrix}$ $(D=)\dfrac{\left\|\left(\begin{pmatrix}0\\0\\18\end{pmatrix}-\begin{pmatrix}0\\11\\7\end{pmatrix}\right)\bullet\begin{pmatrix}8\\-4\\-20\end{pmatrix}\right\|}{\left\|\begin{pmatrix}8\\-4\\-20\end{pmatrix}\right\|}$	M1	3.4	Calculate vector product **n** with $\begin{pmatrix}6\\7\\1\end{pmatrix}$ and the direction vector of their T from part **(a)** with at least one component correct (following through their vectors), **and** setting up the equation $D=\dfrac{\left\|\left(\begin{pmatrix}0\\0\\18\end{pmatrix}-\mathbf{a}\right)\bullet\mathbf{n}\right\|}{\|\mathbf{n}\|}$ where **a** is a point on their T from part **(a)** and their vector **n**. Condone lack of modulus signs in equation. Note that $\pm$ these vectors are valid as are scalar multiples of **n**.
		$=\dfrac{22\sqrt{30}}{15}(=8.033264...)>5$ so yes (the site) passes Test 1.	A1	2.2a	Correct calculation (either exact e.g. $\dfrac{176}{4\sqrt{30}}$) or at least **2** significant figures), including reference to 5 and conclusion.
			[2]		

Question		Answer	Marks	AO	Guidance
	(c)	$(\overrightarrow{PQ}=)\pm\left(\begin{pmatrix}0\\0\\18\end{pmatrix}+\lambda\begin{pmatrix}6\\7\\1\end{pmatrix}-\begin{pmatrix}0\\11\\7\end{pmatrix}\right)\quad\left(=\pm\begin{pmatrix}6\lambda\\7\lambda-11\\\lambda+11\end{pmatrix}\right)$	B1	3.4	**No MR in this part.** For $(0,11,7)$ to a general point on *S*. Allow correct un-simplified. Can be implied by a correct expression for the distance (or distance squared)
		$\left(\left\|\overrightarrow{PQ}\right\|=\right)\sqrt{(6\lambda)^2+(7\lambda-11)^2+(\lambda+11)^2}$	B1	3.4	Expression for $\left\|\overrightarrow{PQ}\right\|$ or $\left\|\overrightarrow{PQ}\right\|^2$ - allow un-simplified.
		$(6\lambda)^2+(7\lambda-11)^2+(\lambda+11)^2\geqslant 19^2\quad(86\lambda^2-132\lambda-119\geqslant 0)$ $\Rightarrow\lambda\geqslant 2.17...$ or $\lambda\leqslant -0.637...$ but $0\leqslant\lambda\leqslant 2$ so no, it's not possible.	B1	3.1b	Solving inequality/equation for λ and concluding no + reason. Condone "no" as conclusion. Critical values of λ correct to at least **2** sf. Allow strict inequalities.
			[3]		
		Alternative method			
		Distance between $(0,11,7)$ and $(0,0,18)$ is $\sqrt{11^2+(7-18)^2}$	B1		Find distance (or distance squared) between (0, 11, 7) and (0, 0, 18) – allow un-simplified.
		Distance between $(0,11,7)$ and $(12,14,20)$ is $\sqrt{12^2+(14-11)^2+(20-7)^2}$	B1		Find distance (or distance squared) between (0, 11, 7) and (12, 14, 20) – allow un-simplified
		$11\sqrt{2}=15.5563...$ and $\sqrt{322}=17.9443...$ which are both less than 19 and the two point (0, 0, 18) and (12, 14, 20) are the points furthest from (0, 11, 7) so no, it's not possible.	B1		Correct values given to at least **3** sf (or in a form in which all three can be compared e.g. $\sqrt{322},\sqrt{242}$ and $\sqrt{361}$) and some indication that these two points are the furthest from the camera and conclude no.
			[3]		

Question	Answer	Marks	AO	Guidance
12	$\frac{1}{2}\int_0^{2\pi}\left(k\left(\cos\theta+1\right)+\frac{10}{k}\right)^2 \mathrm{d}\theta$ or $\int_0^{\pi}\left(k\left(\cos\theta+1\right)+\frac{10}{k}\right)^2 \mathrm{d}\theta$	**M1***	3.1a	Using $\frac{1}{2}\int r^2\mathrm{d}\theta$ with correct expression for r - condone lack of (or incorrect) limits. Condone missing $\frac{1}{2}$ only.
	$=\frac{1}{2}\int_0^{2\pi} k^2\left(\cos^2\theta+2\cos\theta+1\right)+2k\left(\frac{10}{k}\right)(\cos\theta+1)+\frac{100}{k^2}\mathrm{d}\theta$			
	$=\frac{1}{2}\int_0^{2\pi}\frac{k^2}{2}\left(\cos 2\theta+1\right)+2k\left(k+\frac{10}{k}\right)\cos\theta+\left(k+\frac{10}{k}\right)^2\mathrm{d}\theta$	**M1**	1.1	Expanding and using correct $\cos^2\theta=\frac{1}{2}(1+\cos 2\theta)$ - need not be in an integral.
	$=\frac{1}{2}\left[\frac{k^2}{2}\left(\frac{1}{2}\sin 2\theta\right)+2k\left(k+\frac{10}{k}\right)\sin\theta+\left(\frac{k^2}{2}+\left(k+\frac{10}{k}\right)^2\right)\theta\right]_0^{2\pi}$	**M1dep***	1.1	Integrating their $\int k_1\cos 2\theta+k_2\cos\theta+k_3\,\mathrm{d}\theta$ correctly to $\frac{1}{2}k_1\sin 2\theta+k_2\sin\theta+k_3\theta$ with $k_1,k_2,k_3\neq 0$
	$=\frac{k^2}{8}\sin 4\pi+k\left(k+\frac{10}{k}\right)\sin 2\pi+2\pi\left(\frac{k^2}{2}+\left(k+\frac{10}{k}\right)^2\right)$	**M1**	1.1	Substitute correct limits 0 and 2π - dependent on previous **M** mark. Need not see zeros from 0 limit or other terms that would be zero when evaluated.
	$=\pi\left(\frac{3k^2}{2}+20+\frac{100}{k^2}\right)$	**A1**	1.1	cao from **correct integral and correct integration**. Condone $\pi\left(3k^2+40+200k^{-2}\right)$ from missing $\frac{1}{2}$.
	$\frac{\mathrm{d}}{\mathrm{d}k}\left(\pi\left(\frac{3k^2}{2}+20+\frac{100}{k^2}\right)\right)=\pi\left(3k-\frac{200}{k^3}\right)(=0)$	**M1**	2.1a	Differentiating their A_k correctly which must be of the form ak^2+b+ck^{-2} with $a,b,c\neq 0$. Dependent on all previous **M** marks.
	$k=\left(\frac{200}{3}\right)^{\frac{1}{4}}$	**A1**	1.1	Dependent on all previous marks and no errors. **A0** if correct k obtained but $\frac{1}{2}$ missing from area formula (unless justified). Accept any **exact** equivalent. **Condone omission of π (or other constant factor) for final two marks.**
		[7]		

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